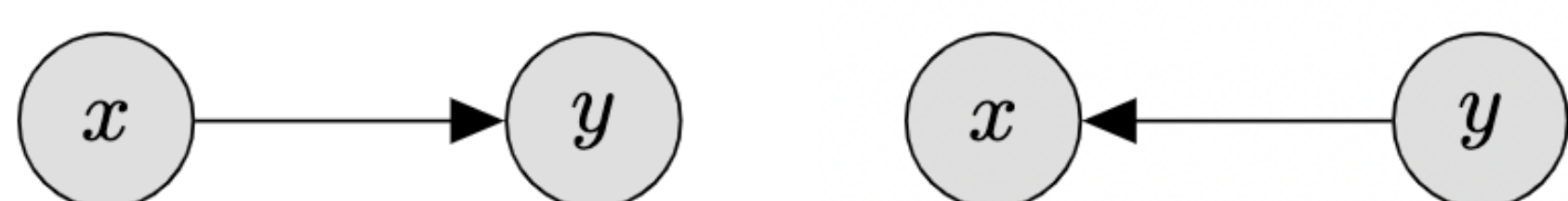


Bayesian model selection allows for flexible discovery of bivariate causal relations

Bivariate Causal Discovery using Bayesian Model Selection

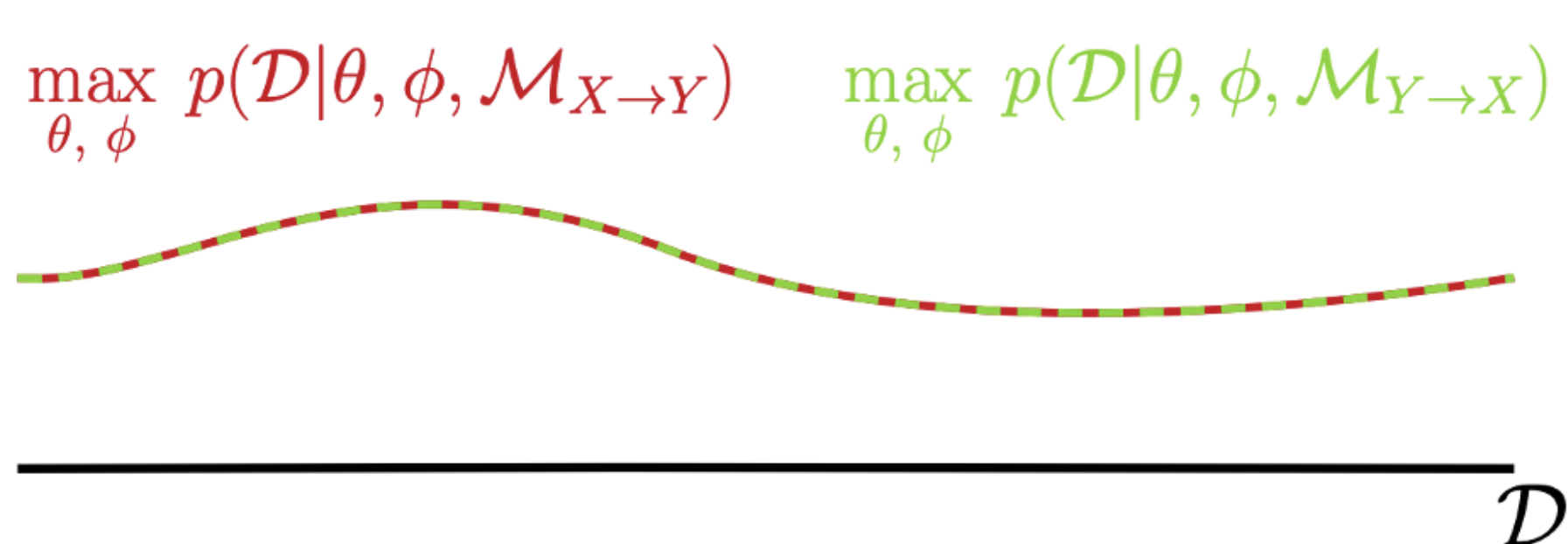
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Introduction

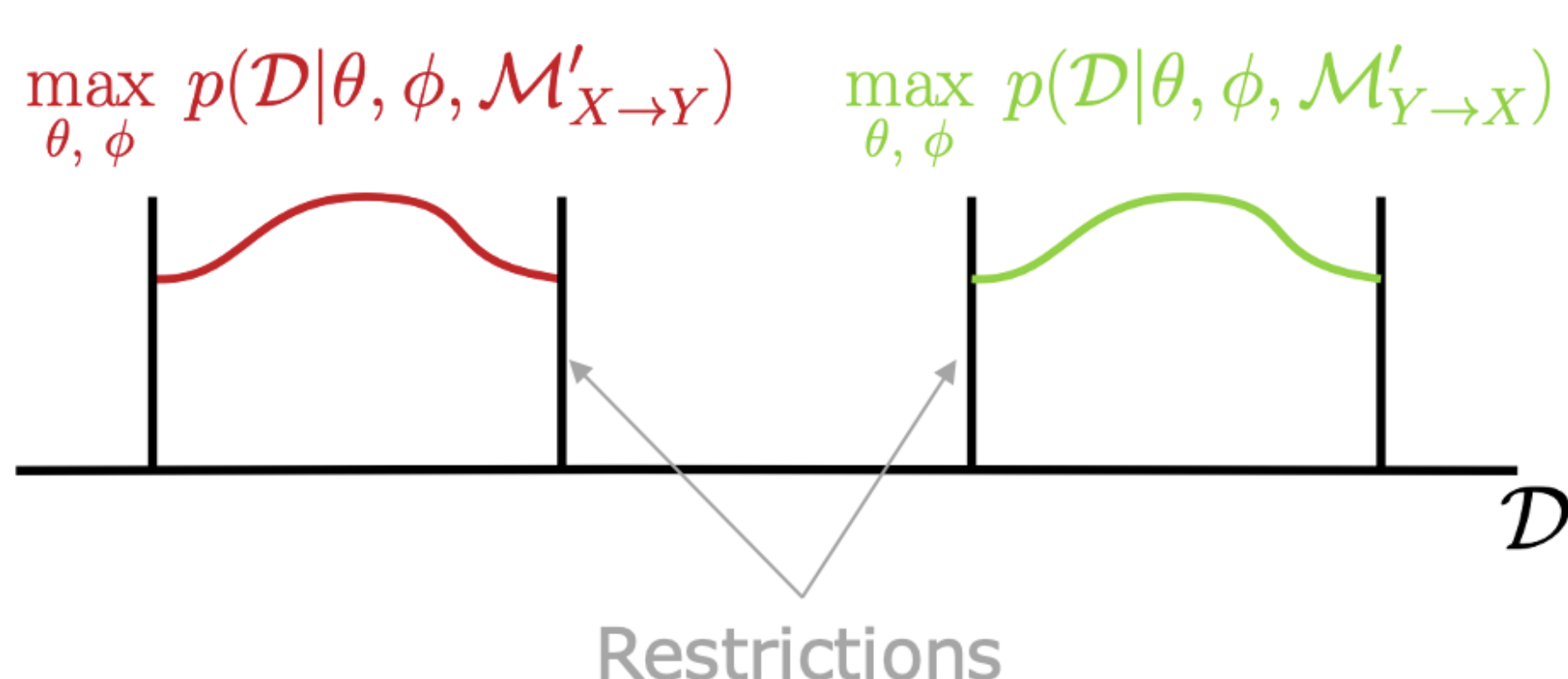


Given variables can we find the causal direction?

Problem: $P(Y|X)P(X) = P(X|Y)P(Y)$.
Maximising likelihood cannot identify the causal direction.



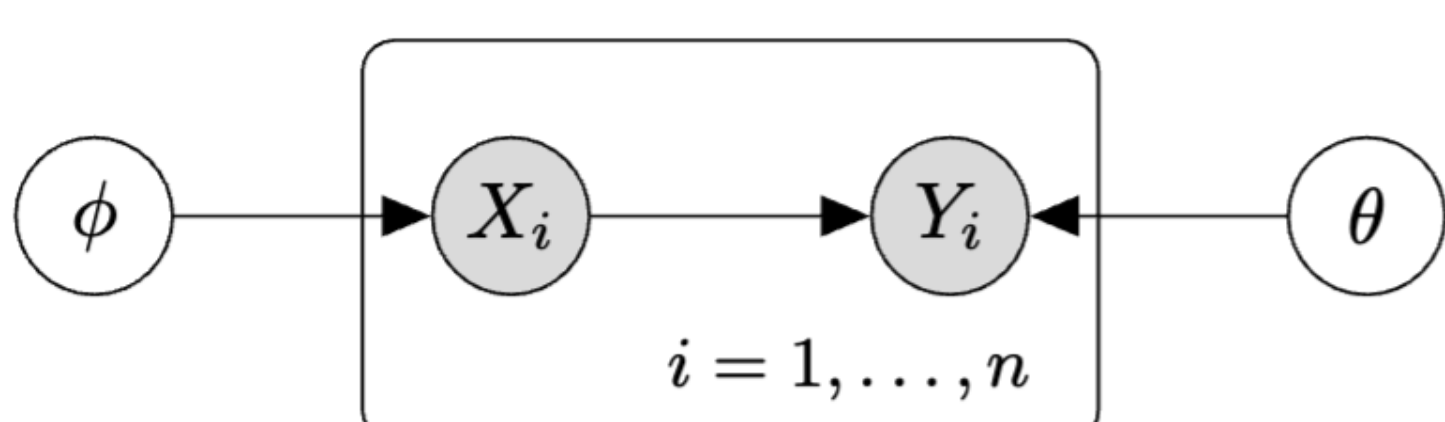
Previous solutions: Restrict model class to allow maximum likelihood to identify, but restricts the datasets you can model!



Idea: Use Bayesian Model selection
Each causal direction is a separate model

$$p(\mathcal{M}_{X \rightarrow Y}|\mathcal{D}) = \frac{p(\mathcal{D}|\mathcal{M}_{X \rightarrow Y})p(\mathcal{M}_{X \rightarrow Y})}{p(\mathcal{D})}$$

$p(\mathcal{D}|\mathcal{M}_{X \rightarrow Y})$ is the **Marginal Likelihood**



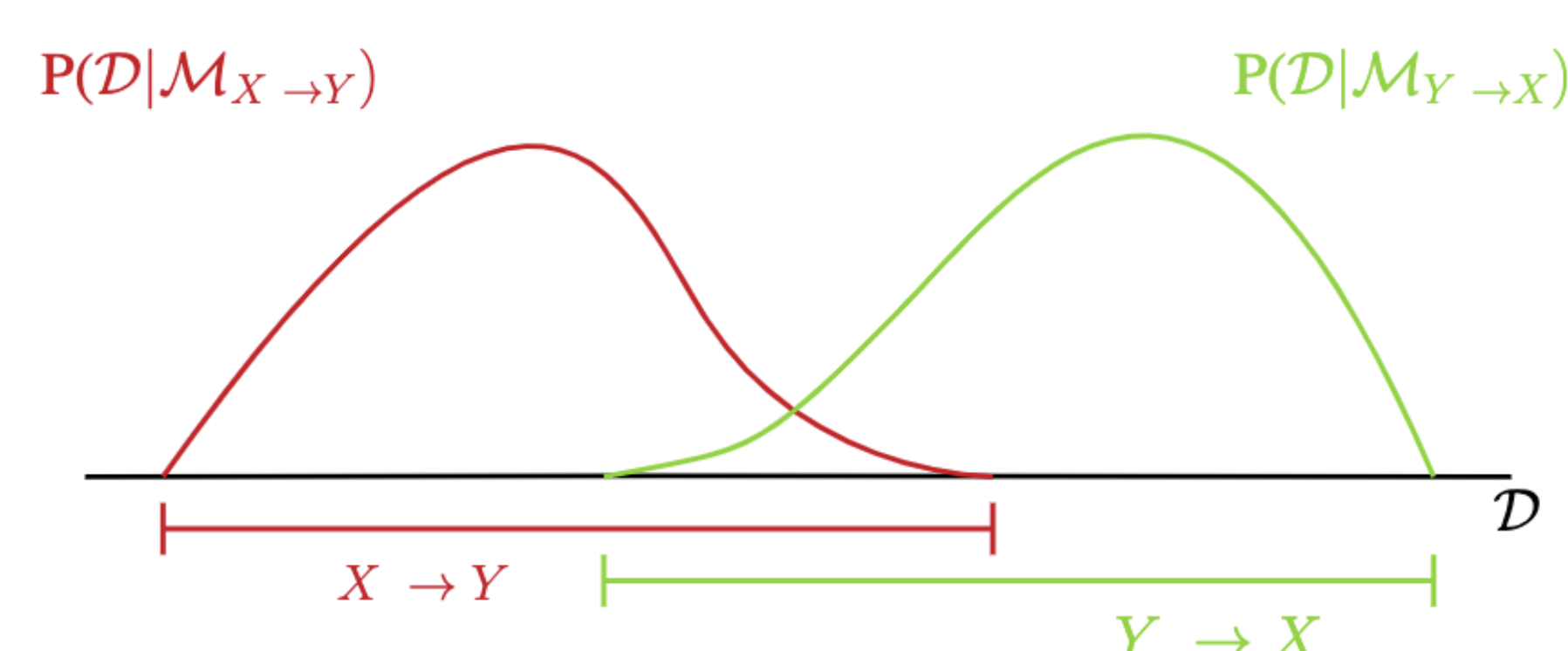
Marginal likelihood $p(x, y|\mathcal{M}_{X \rightarrow Y})$:

$$\int p(y|x, \theta)p(\theta)d\theta \int p(x|\phi)p(\phi)d\phi$$

- Terms in the causal factorisation are parametrised independently.
- Parameters are independent $\theta \perp\!\!\!\perp \phi$
- Assumptions $p(\theta)p(\phi)$ are required

Assumptions:

- Assumptions can reconstruct previous known identifiability with maximum likelihood.
- Can also identify in the case of **flexible** models, where maximum likelihood cannot.
- Marginal likelihood must sum to 1 over datasets. This **restricts** how well our model can explain all datasets.



Correctness:

- Can quantify probability of finding the correct model

$$P(E) = \frac{1}{2} \left(1 - \frac{\text{TV}[P_{\mathcal{D}}(\cdot|\mathcal{M}_{X \rightarrow Y}), P_{\mathcal{D}}(\cdot|\mathcal{M}_{Y \rightarrow X})]}{\text{Total variation between model densities}} \right)$$

- Total variation of 1 corresponds to completely identifiable case
- Under model misspecification our method is not brittle

$$\left| \frac{\prod(\text{Error})}{\text{True probability of error}} - \frac{P(\text{Error})}{\text{Model probability of error}} \right| \leq \text{TV}[\prod_{\mathcal{D}}(\cdot|X \rightarrow Y), P_{\mathcal{D}}(\cdot|\mathcal{M}_{X \rightarrow Y})]$$

GPLVMs

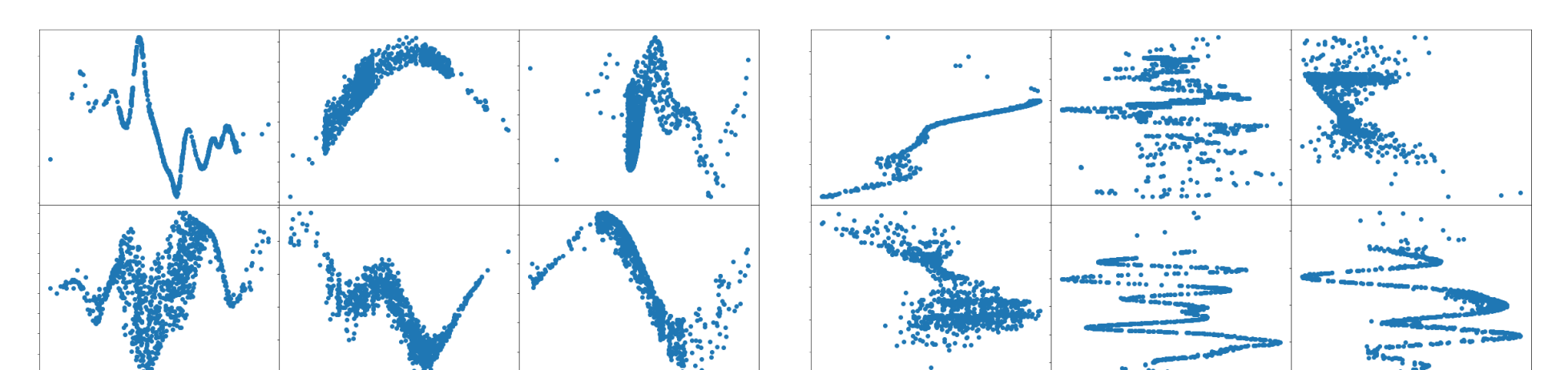
We want to use a **flexible** Bayesian model with the ability to model:

- Non Gaussian likelihoods
- Heteroscedastic noise

We use **Gaussian Process Latent variable models**

$$f(\cdot, \cdot) \sim \mathcal{N}(0, K_{\rho}(\mathbf{x}, \mathbf{w}))$$

$$p(\mathbf{y}|\mathbf{x}, \mathbf{f}) = \int \mathcal{N}(\mathbf{f}(\mathbf{x}, \mathbf{w}), \sigma^2)p(\mathbf{w})d\mathbf{w}$$



Results

Flexibility of our method allows for good performance across a wide range of data generating assumptions (Metric: AUPRC, higher is better)

Methods	Cha	Multi	Net	Gauss	Tueb
LiNGAM	57.8	62.3	3.3	72.2	31.1
ANM	43.7	25.5	87.8	90.7	63.9
PNL	<u>78.6</u>	51.7	75.6	84.7	73.8
IGCI	55.6	77.8	57.4	16.0	63.1
RECI	59.0	94.7	66.0	71.0	70.5
SLOPPY	60.1	95.7	79.3	71.4	65.3
CGNN	76.2	94.7	86.3	89.3	<u>76.6</u>
GPI	71.5	73.8	88.1	90.2	70.6
CDCI	72.2	<u>96.0</u>	<u>94.3</u>	91.8	58.7
GPLVM	81.9	97.7	98.9	89.3	78.3



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